Dispersion Compensation for Atom Interferometry

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A new technique for maintaining high contrast in an atom interferometer is used to measure large de Broglie wave phase shifts. Dependence of an interaction induced phase on the atoms' velocity is compensated by applying an engineered *counterphase*. The counterphase is equivalent to a rotation, is precisely determined by a frequency, and can be used to measure phase shifts due to interactions of unknown strength. Phase shifts of 150 rad (5 times larger than previously possible) have now been measured in an atom beam interferometer, and we suggest that this technique can enable comparisons of atomic polarizability with precision of one part in 10 000.

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Atom interferometers are now precision tools for measuring interactions that cause a differential phase shift between atom waves in two separated paths. For example, measurements of platform rotation [1,2] and acceleration [3,4], gravity gradients [5], and atomic polarizability [6] have each been made using atom interferometers to directly measure a corresponding atomic de Broglie wave phase shift. The precision of these measurements made with cw atom beams can be improved by compensating for dispersion. *Dispersion,* i.e., a correlation between atom wavelength and phase shift, has limited the interferometer in [1,6] to a maximum of 35 rad of interaction induced phase shift before contrast is reduced by $1/e$. In Ref. [2] a rotation induced phase of 10 rad could reduce the contrast by $1/e$. Here we present a new technique to maintain high contrast while studying large de Broglie wave phase shifts without reducing atom flux. It consists of two separated regions that induce time-dependent phases in a way that the net applied phase depends on the atom's velocity. We shall describe this technique and show that it is equivalent to rotation at an accurately known angular frequency.

The source of contrast loss addressed here comes from the experimental spread in atomic velocity combined with dispersion. Most interactions are dispersive because the interaction induced phase shift, or *interaction phase,* depends on velocity to some power: $\phi_{\mathrm{int}}(v) \propto v^n$. The factor n equals -1 for interactions with phase shifts that depend on transit time, such as platform rotation or uniform fields applied to one arm of an interferometer. The factor n equals -2 for gravitationally induced phase shifts and also for electric or magnetic field gradients across an interferometer made with gratings. For $n \neq 0$, a spread in velocity leads to an inhomogeneous phase and hence a loss of contrast. Under such circumstances the statistical power in a measurement of interaction strength is optimized at a rather small interaction phase. For a Gaussian atomic velocity distribution, this phase is $\phi_{\mathrm{int}}(v_0) = |\frac{1}{n}|\frac{v_0}{\sigma_v}$, where v_0 is the average and σ_v is the rms width of the velocity distribution. The supersonic atom beam in Ref. [1,6] has $\frac{v_0}{\sigma_v} = 25$, which limits the most sensitive measurements to an interaction phase of 25 rad.

Dispersion compensation enables measurements of much larger interaction induced phase shifts. We demonstrate this by using an engineered *counterphase* to cancel dispersion, and regain high contrast. The technique has many advantages for precision measurements of an interaction strength — a very large interaction phase can now be measured, all the atom flux is used, and the need to precisely measure the velocity of the atoms is eliminated. It is a quantum extension of the classical velocity multiplexing technique that used mechanical choppers to modify the velocity distribution of the beam [7].

In an earlier proposal, Clauser [8] noted that a magnetic field gradient can compensate for gravitationally induced phase shifts. This idea, in essence, allows for measuring one interaction in terms of another, the overall error being a combination of the two errors separately. In comparison, the counterphase used here is determined by a frequency that can be set to a precisely known and stable value. Our technique [9] is more closely related to methods developed in [10] and used in [2–5] where dispersion compensation is achieved by moving the gratings to simulate platform rotation or acceleration.

To create the counterphase, two phase shift regions spaced a distance L_{shifters} apart are used to produce differential phase shifts between the arms of the interferometer (Fig. 1). They apply a saw-tooth ramp — increasing the applied phase linearly from zero to $\pm 2\pi$ and then abruptly returning to zero — with frequency f. The phase shifts must be opposite in sign, as shown in Fig. 2, and can be represented, modulo 2π, as

$$\phi_1(t) = +2\pi f t, \qquad \phi_2(t) = -2\pi f t. \tag{1}$$

The sum of these phase shifts makes the counterphase

$$\begin{aligned}\phi_{\mathrm{counter}}(t, v) &= \phi_1(t) + \phi_2(t + L_{\mathrm{shifters}}/v)\\ &= -2\pi f L_{\mathrm{shifters}}/v.\end{aligned} \tag{2}$$

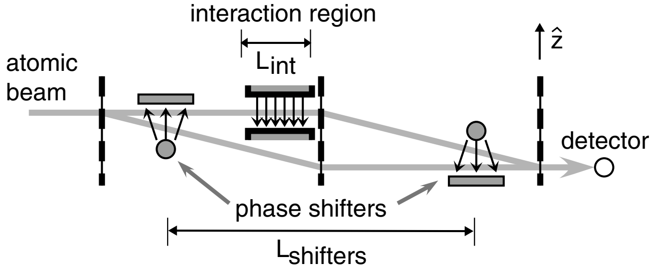

FIG. 1. Schematic of the three-grating interferometer. An interaction region is in the center, and the two phase shifters used for dispersion compensation are located on either side.

Importantly, the counterphase depends on the atom's time of flight between the two phase shifters; i.e., it has v^{-1} dispersion, but is independent of time. The dispersion increases with ramp frequency; hence, it can compensate for a dispersive interaction of any strength. Fidelity of the two phase shifts $\phi_{1,2}$ to Eq. (1) becomes less critical when the ramp frequency is large as is discussed later.

The cancellation of dispersive effects in principle works perfectly if the interaction phase is proportional to v^{-1}. One such interaction is a region of uniformly different potentials of length L_{int} on one interferometer arm. The change in energy, $\hbar\omega_{\text{int}}$, for an atom inside the field is the same for atoms of all velocities, but the phase shift caused by the interaction depends on the transit time L_{int}/v of the atom passing through the region:

$$\phi_{\text{int}}(v) = \omega_{\text{int}} L_{\text{int}}/v. \tag{3}$$

With this interaction alone, a spread in velocity from the atom source creates a spread in ϕ_{int} that destroys the contrast of the interference fringes and limits the size of ω_{int} that can be measured. By adding the counterphase, the total phase shift is

$$\phi_{\text{int}}(v) + \phi_{\text{counter}}(v) = (\omega_{\text{int}} L_{\text{int}} - 2\pi f L_{\text{shifters}})/v. \tag{4}$$

At the rephasing frequency, $f_{\text{reph}} \equiv \omega_{\text{int}} L_{\text{int}}/2\pi L_{\text{shifters}}$, the total phase shift is zero for all velocities. There is no net dispersion, and the fringe contrast should be ideal.

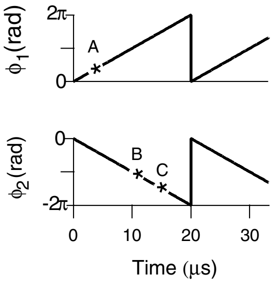

FIG. 2. The time-dependent phases, $\phi_1(t)$ and $\phi_2(t)$, introduced by each phase shifter. The sum $\phi_1(t) + \phi_2(t + L_{\text{shifters}}/v)$ depends on velocity. For example, faster atoms may get phase shifts A and B, while slower atoms get the phase shifts A and C.

This method of rephasing the interference pattern can improve the contrast in any atom beam or atom chip interferometer which is affected by dispersion. The example we present here is a dramatic improvement in a precision measurement of atomic polarizability such as was made with the MIT interferometer [6]. We can now apply an interaction phase shift exceeding 150 rad for the best signal-to-noise ratio, compared with 25 rad previously. Furthermore, instead of measuring the atom beam velocity (and velocity distribution) and then modeling the phase shift, we need to measure only the ramp frequency and the distance L_{shifters}. In this proof-of-principle experiment, these advantages made it possible to determine the polarizability of sodium atoms with better statistical precision than in Ref. [6] but in 1/15 the time.

We have implemented the phase shifters by using small regions of the electric field gradient produced by a charged cylinder with radius $r = 0.5$ mm at a voltage of $V_0 \approx 2$ kV. The paths of the interferometer pass between the cylinder and a ground plane, which is a distance $a = 1.5$ mm from the cylinder axis (Fig. 3). The phase difference between the paths (separated by distance $w \approx 50$ μm) is

$$\phi_{1,2}(t) \approx \pm \frac{\pi}{2} \ln^{-2}\left(\frac{2a}{r}\right) \frac{\alpha w}{v x^2} V_0(t)^2, \tag{5}$$

where x is the average distance of the two paths from the cylinder axis and α is the polarizability of the atoms. The two cylinders are oriented on opposite sides of the interferometer so that they can apply opposite relative phases as required by Eq. (1). To create a linear ramp in phase, a voltage must be applied to the cylinders with time dependence $V_0(t) \sim \sqrt{t}$. We approximate this ideal square-root shape in time by filtering a high voltage rectangle wave (with duty cycle $p \approx 90\%$) using an RC circuit. During the off cycle a diode allows the capacitor voltage to quickly return to zero. Using this voltage waveform, the phase produced by the gradient fields ramps

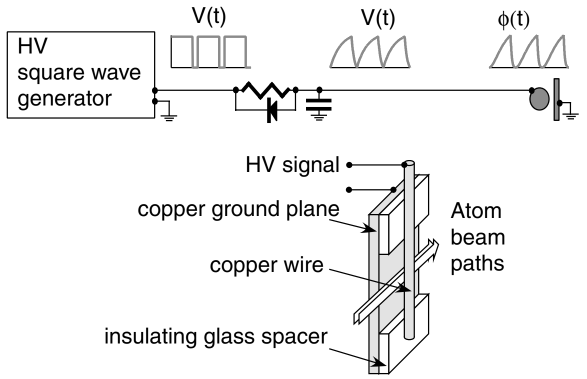

FIG. 3. A schematic of the electronics driving each phase shifter, including a blowup showing how the gradient field region is constructed. Note that $\phi_1 \sim V^2(t)$ is a good approximation to a linear sawtooth.

approximately linearly from 0 to 2π during the on cycle:

$$\phi_{1,2}(t) = \pm\gamma(1 - e^{-t/RC})^2 \quad \text{for } 0 < t < p/f. \tag{6}$$

The parameter γ, proportional to the strength of the gradient field, can be changed by adjusting the amplitude of the square wave or the position x of each cylinder. Setting $\gamma = 0.83\pi$ rad and $RC = 2.4/f$ best approximates the perfect ramp defined by Eq. (1). High fidelity to Eq. (1) is not required if the ramp frequency is much larger than the inverse time of flight between the two phase shifters, since the deviation in phase is small compared to the additional phase of $2\pi m$ corresponding to m ramp cycles.

Applied to the measurement of polarizability, the phase shifters are used in conjunction with an interaction phase produced by a parallel plate capacitor (with voltage difference V and plate separation d) that makes a constant electric field surrounding one path of the interferometer for a length L_{int}. The change in energy of an atom in the field is given by the polarizability α:

$$\hbar\omega_{\text{int}} = \tfrac{1}{2}\alpha V^2/d^2. \tag{7}$$

To determine α, the phase-shifted interference pattern is measured as oscillating atom beam intensity versus the transverse position z of one grating. For atoms of velocity v, the interference pattern without a counterphase is

$$I_v(z) = N + A\cos[k_g z + \phi_{\text{int}}(v)], \tag{8}$$

where N is the average intensity, A is the amplitude of the fringe, and $k_g = 2\pi/(100\text{ nm})$ is the grating wave number. The measured interference pattern is a weighted average over the velocity distribution (which is approximately Gaussian with $\sigma_v/v_0 \approx 0.04$, where v_0 is the average velocity and σ_v is the rms velocity width):

$$\begin{aligned} I(z) &= \int dv P(v)(N + A\cos[k_g z + \phi_{\text{int}}(v)]) \\ &= N + AC'\cos[k_g z + \phi'], \end{aligned} \tag{9}$$

where the resulting phase and contrast are

$$\phi' = \phi_{\text{int}}(v_0), \qquad C' = \exp\left[-\frac{1}{2}\left(\frac{\sigma_v}{v_0}\right)^2 \phi^2_{\text{int}}(v_0)\right]. \tag{10}$$

With both the counterphase and interaction phase, the resulting interference pattern has

$$\begin{aligned} \phi' &= \phi_{\text{int}}(v_0) - 2\pi f L_{\text{shifters}}/v_0, \\ C' &= \exp\left[-\frac{1}{2}\left(\frac{\sigma_v}{v_0}\right)^2\left(\phi_{\text{int}}(v_0) - \frac{2\pi f L_{\text{shifters}}}{v_0}\right)^2\right]. \end{aligned} \tag{11}$$

Note C' is merely a shifted Gaussian that has the same width independent of ramp frequency. In principle, the peak contrast should remain $C' = 1$.

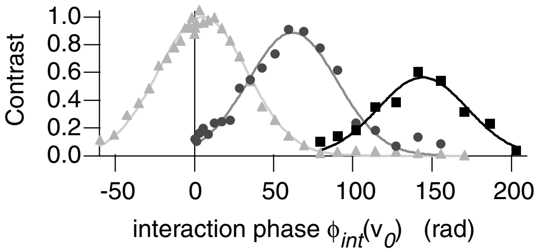

FIG. 4. Contrast of the interference pattern as a function of interaction phase, $\phi_{\text{int}}(v_0)$. With no dispersion compensation (▲) contrast peaks near $\phi_{\text{int}}(v_0) = 0$. Using the counterphase, contrast revives at $\phi_{\text{int}}(v_0) = 62$ rad for a ramp frequency $f =$ 17 kHz (●) and at $\phi_{\text{int}}(v_0) = 144$ rad for $f = 40$ kHz (■).

Measurements of contrast vs interaction phase (Fig. 4) were made with the phase shifters ramping at fixed frequencies. A revival in contrast occurs when $\phi_{\text{int}}(v_0) = 2\pi f L_{\text{shifters}}/v_0$. However, the maximum rephased contrast is less than the ideal 100% because of imperfections discussed in [9] such as the width of the atom beam, the implemented phase ramp and the finite length of the phase shifter regions. The rightmost curve (■) in Fig. 4 demonstrates that polarizability can now be measured at an interaction phase as large as 150 rad.

Polarizability was measured using this technique by finding the parameters at which the total phase is exactly zero, at which point there is high contrast and no explicit dependence on velocity. Phase measurements are shown in Fig. 5 and are fit to a line whose zero crossing determines V^2_{reph}. The polarizability is determined by

$$\alpha = \frac{2hfL_{\text{shifters}}d^2}{L_{\text{int}}V^2_{\text{reph}}}. \tag{12}$$

The precision of the polarizability measurement depends on the uncertainty $\Delta\phi'$ with which the interaction phase can be measured, and also on $\phi_{\text{int}}(v_0)$:

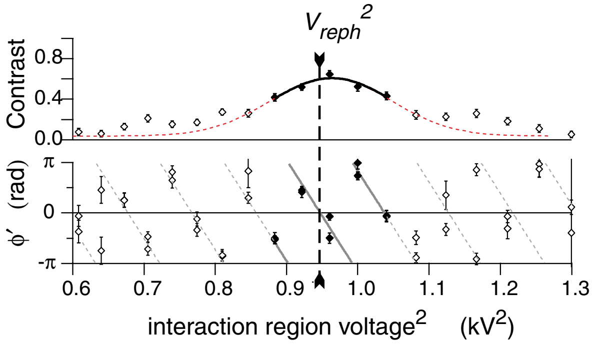

FIG. 5 (color online). The relative contrast and phase as a function of the voltage squared in the interaction region. Atomic polarizability can be determined by finding the voltage, V^2_{reph}, at which the fit to the total phase, ϕ', intercepts zero (and the contrast is nearly maximum). The fit was made only to the 10 central data points (◆) (two at each voltage). The dashed line is an extrapolation from the fit, shown for comparison with the data outside the fit region (◇).

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta\phi'}{\phi_{\rm int}(v_0)}. \tag{13}$$

Without the counterphase [e.g., in the leftmost curve (▲) of Fig. 4] the best signal-to-noise ratio occurs at an interaction phase of $\phi_{\rm int}(v_0) = v_0/\sigma_v = 25$ rad. Roughly 300 sec of measurement time is needed to achieve a precision in polarizability of $\Delta\alpha/\alpha = 0.2\%$.

With the counterphase we achieve the same statistical precision in α with only 20 sec of measurement. The ten phase measurements used in the linear fit (◆ data points in Fig. 5) constitute a total of 20 sec of measurement, and the resulting uncertainty in the interaction phase is $\Delta\phi' = 130$ mrad, while the interaction phase itself is $\varphi_{\rm int}(v_0) = 66$ rad. Thus, the same fractional uncertainty in polarizability of 0.2% is achieved in 1/15 the time. Using the contrast revival demonstrated at 150 rad, it should be possible to measure the interaction phase to 10^{-4} statistical precision with 50 min of data.

Sources of systematic error include uncertainty in $L_{\rm shifters}$, the calibration of the phase shifts $\phi_{1,2}$, and the dimensions of the interaction region. (The frequency f can be determined essentially without errors.) The most difficult new systematic errors to analyze stem from deviations in the ramped phase from the ideal form in Eq. (1). If the ramp maxima deviate from $\pm 2\pi$ by ϵ but are symmetric, i.e. $\phi_1(t) = -\phi_2(t)$, the corresponding fractional error in α is smaller than $\epsilon/2\pi$ by a factor of $L_{\rm shifters} v^{-1} f^{-1}$ because each time $\phi_2(t)$ returns to zero during an atom's time of flight, it is equivalent to adding a precise integer times -2π to the counterphase. However, if $\phi_1(t) \neq -\phi_2(t)$, the error in phase is

$$\phi_{\rm error} = \langle \phi_1(t) + \phi_2(t) \rangle_t. \tag{14}$$

To correct for this, $\phi_{\rm error}$ was measured at $f = 0$ [9].

The dispersion compensation technique presented here eliminates two of the three major sources of uncertainty that limited the precision in α obtained in [6] (i.e., contrast loss and uncertainty in the velocity distribution). However, a measurement of sodium's polarizability at the 10^{-4} level is still difficult due to uncertainty in the geometry of the interaction region. To attain 10^{-4} precision in the parameter $d^2/L_{\rm int}$ in Eq. (12), the dimensions must be measured to within 10^{-7} m and the electric field modeled very accurately. New techniques can meet these requirements [9]. However, by reporting the *ratio* of polarizabily of two atoms measured with the same apparatus, all the geometrical lengths ($L_{\rm shifters}$, $L_{\rm int}$, and d) cancel so the resulting ratio is limited only by statistical errors if the atoms traverse the same path through the interaction region.

Our dispersion compensation technique is also useful for Sagnac gyroscopes because rotations create a $1/v$-dependent phase shift,

$$\phi_{\rm rotation}(v) = 2k_g L_g^2 \Omega/v, \tag{15}$$

where Ω is the rotation rate and L_g is the distance between the interferometer's gratings, and k_g is the wave number of the gratings [1,2]. Our technique exactly cancels such dispersion and may therefore be regarded as equivalent to physically rotating the interferometer. For example, in Figs. 4 and 5 the contrast C' is maximized at $\phi' = -\phi_{\rm rotation}(v_0)$ with Ω determined by the earth's rotation rate. Thus, one could servo the engineered phase and use the integral of the rephasing frequency as a measurement of accumulated angular displacement.

The rephasing technique also works to some extent for interactions with *any* velocity dependence, $\phi_{\rm int}(v) \propto v^n$. For example, the interaction phase with an optimum signal-to-noise ratio can be increased by the factor $v_0/|n+1|\sigma_v$ since

$$\phi_{\rm int}(v_0) < \left|\frac{1}{n(n+1)}\right| \left(\frac{v_0}{\sigma_v}\right)^2 \tag{16}$$

is allowed by using the v^{-1} counterphase presented here.

In conclusion, we have demonstrated a new method to maintain high contrast of the interference pattern while studying large de Broglie wave phase shifts in an atom interferometer. The method is of immediate benefit to new precision measurements of atomic polarizability. In addition, the technique serves to restore contrast in any atom interferometer that is subject to velocity dependent phase shifts.

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